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BSc6MatC602x

Seat No :________

B.Sc. Semester - 6 (CBCS) Examination

May/June-2021 (NEW COURSE)

Mathematical Analysis -2 and Abstract Algebra -2(Core)

Time: 1:30 Hours

Marks: 42

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1(A) **Answer any two:** (10)

(1) Let (X, d) be metric space and E_1 and E_2 are connected subsets of X. If $E_1 \cap E_2 \neq \phi$ then prove that $E_1 \cup E_2$ is also connected subset of X.

(2) State and prove theorem of nested intervals.

(3) Prove that a metric space (X, d) is sequentially compact iff X has Bolzano-Weirstrass property.

Q.1(B) **Answer any one:** (04)

(1) Define: Compact metric space. If (X, d) is compact metric space then prove that X is totally bounded set.

(2) Give examples of two disjoint sets which are not separated and prove your claim.

Q.2(A) **Answer any two:** (10)

(1) Define: Laplace transform. Prove that

(i) $L(sinat) = \frac{a}{s^2+a^2}$ (ii) $L(coshat) = \frac{s}{s^2-a^2}; s > |a|$.

(iii) $L(t^n) = \frac{n!}{s^{n+1}}; n = 0,1,2, \dots..$

(2) Define: Inverse Laplace transform. Find: $L^{-1}\left\{\frac{s^3}{s^4-a^4}\right\}$.

(3) State and prove the first shifting theorem of Laplace transforms and find (i) $L[e^{-3t}(2cos5t - 3sin5t)]$ (ii) $L[t^2 sinht]$.

Q.2(B) **Answer any one:** (04)

(1) Find the Laplace transform of

(i) $cosh^3 2t$ (ii) $e^{-3t} sin^2 t$

(2) Find: (i) $L^{-1}\left\{\frac{8-6s}{16s^2+9} - \frac{3+4s}{9s^2-16}\right\}$ (ii) $L^{-1}\left\{\frac{3(s^2-1)^2}{2s^5}\right\}$

Q.3(A) **Answer any two:** (10)

(1) Derive the formula for finding Laplace transform of the derivative of $f(t)$. Find the Laplace transform of

(i) $\frac{e^{-at}-e^{-bt}}{t}$ (ii) $te^{2t}cos3t$

(2) If $L\{f(t)\} = \bar{f}(s)$ and $\frac{f(t)}{t}$ has Laplace transform then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_0^\infty \bar{f}(s)ds$. Find (i) $L\left(\frac{cos2t-cos3t}{t}\right)$ (ii) $L\left(\frac{sint}{t}\right)$

(3) Using convolution theorem, find

(i) $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ (ii) $L^{-1}\left\{\frac{1}{(s-1)(s^2+1)}\right\}$.

Q.3(B) **Answer any one:** (04)

(1) If $L\{f(t)\} = \bar{f}(s)$ then prove that $L\left[\int_0^t f(u)du\right] = \frac{1}{s}\bar{f}(s)$. Find the Laplace transform of (i) $t^2 sin4t$ (ii) $\frac{1-e^t}{t}$.

(2) Evaluate: (i) $L^{-1}\left[log\left(1 + \frac{4}{s^2}\right)\right]$ (ii) $L^{-1}\left[\frac{1}{s(s^2+4)}\right]$.

Q.4(A) **Answer any two:** (10)

(1) Define: Homomorphism of groups. If $\varphi: (G,*) \to (G', \Delta)$ is homomorphism then prove that

(i) If N is a normal subgroup of G then $\varphi(N)$ is a normal subgroup of $\varphi(G)$.

(ii) If $H' \leq G'$ then $\varphi^{-1}(H') \leq G$.

(2) Prove that the commutative ring R with unity is a field if it has no proper ideal.

(3) Show that $(C[0,1], +, \cdot)$ is a commutative ring with unity, where $C[0,1]$ = the collection of all real valued continuous functions defined on $[0,1]$ and $\forall f, g \in C[0,1]$, $(f+g)(t) = f(t) + g(t), (fg)(t) = f(t)g(t); \forall t \in [0,1]$.

Q.4(B) **Answer any one:** (04)

(1) Show that $\varphi: G \to G, \varphi(x) = x^3, \forall x \in G$ is a homomorphism Where $G = (C, \cdot)$ with $K_\varphi = \{1, w, w^2\}$, where w is cube root of unity.

(2) Define: Left and right ideal. Give an example of right ideal which is not left ideal.

Q.5(A) **Answer any two:** (10)

(1) Define: Degree of a polynomial. Prove that in usual notations, If $f, g \in D[x] - \{0\}$then $[fg] = [f] + [g]$.

(2) If $f(x), g(x) \in F[x] - \{0\}$ then show that there exists their g.c.d. $d(x)$ and $d(x) = a(x)f(x) + b(x)g(x)$, for some polynomials $a(x), b(x)$ of $F[x]$.

(3) State and prove division algorithm for polynomials.

Q.5(B) **Answer any one:** (04)

(1) Find g.c.d. of $f(x) = 6x^3 + 5x^2 - 2x + 25$ and $g(x) = 2x^2 - 3x + 5 \in R[x]$ and express it in the form $a(x)f(x) + b(x)g(x)$

(2) Define: Inverse of quaternion. Simplify:

(i) $(1 + 2j - 3k)^{-2}$ (ii) $(1 + i + k)^{-1}(1 - j + k)$

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